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CONCERNING SOME EXTENSIONS OF S4

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In my papers [9], [11] and [12] several problems concerning some extensions of S4 are left open.* Namely:

(A) In [9], pp. 355-359, sections **2.6** and **2.7**, it has been proved about the modal formula

T1 $\mathfrak{C}\mathfrak{C}\mathfrak{C}pqqC\mathfrak{C}\mathfrak{C}Npqqq$

which was observed by Grzegorczyk in [1], p. 128: (i) that, in the field of S4, **T1** implies **J1**, i.e., the proper axiom of K1.1, *cf.* [10] and [9], p. 349; (ii) that **T1** is a consequence of K1.2, *cf.* [10] and [9], p. 349; (iii) and that **T1** is verified by characteristic matrix which, *cf.* [2], Makinson has constructed for his system D*, i.e., for my system K3.1, *cf.* [5].

But I was able neither to prove logically that **T1** is a consequence of K1.1 nor to establish that the systems K1.1.1 (= {S4; **T1**}) and K2.2 (= {K2; **T1**}), *cf.* [9], p. 367, are the proper extensions of K1.1 and K2.1 respectively.

(B) As Geach has observed, *cf.* [4], p. 139, and [11], p. 305, in the field of S4.2, the so-called Diodorean modal formulas **N1** and **M1** are inferentially equivalent. Although, clearly, in the field of S4, {**M1**} → {**N1**}, up to now it was unknown whether, in the field of the same system, {**N1**} → {**M1**}. Consequently, the problems whether S4.1 (= {S4; **N1**}) and S4.1.2 (= {S4; **L1**; **N1**}) are properly contained in S4.1.1 (= {S4; **M1**}) and in S4.1.3 (= {S4; **L1**; **M1**}) respectively remained open, *cf.* [11], p. 311, and [12].

(C) In [9], pp. 363-366, sections **3.4-3.6**, it has been proved that the system S4.7 which Schumm has established in [6], contains S4.6 (= {S4; **S1**}) which in its turn contains S4.5 (= {S4; **E1**} ⇄ {S4; **E2**}). Moreover, it has been

*An acquaintance with the papers cited in this note and especially, with the enumeration of the extensions of S4 and their proper axioms introduced in [9], pp. 347-350, and in [12], is presupposed.

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