Notre Dame Journal of Formal Logic
Volume XII, Number 3, July 1971
NDJFAM

SOLUTIONS TO FOUR MODAL PROBLEMS OF SOBOCIŃSKI

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In Sobociński's [1], [2], and [3] several questions are left open, among them

Q1. Is K1.1.1 a proper extension of K1.1?
Q2. Is K2.2 a proper extension of K2.1?
Q3. Is S4.1.1 a proper extension of S4.1?
Q4. Is S4.1.3 a proper extension of S4.1.2?

All four questions are here settled in the negative, a familiarity with these three papers being presupposed. We shall also assume a nodding acquaintance with Kripkean relational models and with the fact that α is a thesis of S4 if and only if for every S4 model $\mathfrak{F} = (\mathfrak{G},\mathfrak{K},\mathfrak{R})$, i.e., in which $\mathfrak{R}$ is a reflexive and transitive relation on $\mathfrak{K}$, $\phi(\alpha,\mathfrak{G}) = 1$ for each valuation ϕ on $\mathfrak{F}$.

Ad Q1 and *Q2*. We shall show that

$$CLCLCLCCpLqLCpLqCpLqCpLqCLCLCpLqLqCLCLCNpLqLqLq$$

is validated by every S4 model and thus is a thesis of S4, from which it follows that Grzegorczyk's axiom $CLCLCpLqLqCLCLCNpLqLqLq$ is a thesis of K1.1 and K2.2.

Suppose ϕ is a valuation on an S4 model $(\mathfrak{G},\mathfrak{K},\mathfrak{R})$ such that

$$\phi(CLCLCpLqLqCLCLCNpLqLqLq,\mathfrak{G}) = 0,$$

from which

$$\phi(LCLCpLqLq,\mathfrak{G}) = 1 \quad (1)$$
$$\phi(LCLCNpLqLq,\mathfrak{G}) = 1 \quad (2)$$
$$\phi(Lq,\mathfrak{G}) = 0. \quad (3)$$

The task is now to show that

$$\phi(LCLCLCCpLqLCpLqCpLqCpLq,\mathfrak{G}) = 0, \quad (4)$$

and this will complete the proof. From (1) we get

$$\phi(CLCpLqLq,\mathfrak{G}) = 1$$

and from this, together with (3),

Received October 12, 1970