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A NOTE ON AN AXIOM-SYSTEM OF ATOMISTIC MEREOLOGY

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In [2] and [3]* the system of atomistic mereology with "el" as its single primitive functor is based on two axioms. Namely,

$A \quad [AB] \therefore A \varepsilon \mathbf{el}(B) . \equiv : : B \varepsilon B : : [Ta] : : [C] \therefore C \varepsilon T . \equiv : [B] : B \varepsilon a . \supset . B \varepsilon \mathbf{el}(C) : [B] : B \varepsilon \mathbf{el}(C) . \supset . [\exists EF] . E \varepsilon a . F \varepsilon \mathbf{el}(E) . F \varepsilon \mathbf{el}(B) \therefore B \varepsilon \mathbf{el}(B) . B \varepsilon a . \supset . A \varepsilon \mathbf{el}(T)$

which is Lejewski's single axiom of general mereology, *cf.* [2], section **2**, and the additional atomistic axiom:

$V \quad [A] : : A \varepsilon A . \supset \therefore [\exists B] \therefore B \varepsilon \mathbf{el}(A) : [C] : C \varepsilon \mathbf{el}(B) . \supset . C = B$

Since in the field of general mereology the following formula which is shorter than axiom A:

$B \quad [AB] \therefore A \varepsilon \mathbf{el}(B) . \equiv : : B \varepsilon B : : [Ta] : : [C] \therefore C \varepsilon T . \equiv : [B] : B \varepsilon a . \supset . B \varepsilon \mathbf{el}(C) : [B] : B \varepsilon \mathbf{el}(C) . \supset . [\exists EF] . E \varepsilon a . F \varepsilon \mathbf{el}(E) . F \varepsilon \mathbf{el}(B) \therefore B \varepsilon a . \supset . A \varepsilon \mathbf{el}(T)$

holds, as an inspection of the proofs of *P10* and *P11* from [3], section **4.2**, can show easily, an occurrence of a subformula "$B \varepsilon \mathbf{el}(B)$" in A is rather irritating. But, up to now any endeavor to substitute A by B, as a single axiom of mereology, failed. In this note it will be proved that in the axiom-system of atomistic mereology which is presented above axiom A can be substituted by B.

Proof: Let us assume B and V. Then:

$A1 \quad [AB] : A \varepsilon \mathbf{el}(B) . \supset . B \varepsilon B \qquad [B]$

$Z1 \quad [ABa] \therefore B \varepsilon a : [B] : B \varepsilon a . \supset . B \varepsilon \mathbf{el}(A) : \supset . A \varepsilon A \qquad [A1]$

$D1 \quad [Aa] \therefore A \varepsilon A : [B] : B \varepsilon a . \supset . B \varepsilon \mathbf{el}(A) : [B] : B \varepsilon \mathbf{el}(A) . \supset . [\exists EF] . E \varepsilon a . F \varepsilon \mathbf{el}(E) . F \varepsilon \mathbf{el}(B) : \equiv . A \varepsilon \mathbf{Kl}(a)$

*An acquaintance with [2] and [3] is presupposed. An enumeration of the theorems which are appearing in this note, except for *B*, *Z1*, *Z2* and *Z3*, is the same as in those papers.

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