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DUALITY IN FINITE MANY-VALUED LOGIC

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1. *Introduction* The notion of duality is a familiar one in the two-valued propositional calculus. In view of the one-many correspondence between a truth-function and well-formed formulas (proposition-letter formulas), a well-formed formula can have many different duals—the well-formed formulas that have the truth-table of the dual truth-function. We can obtain a dual (called the "principal dual" in Church [1]) of a well-formed formula in a mechanical way by replacing each occurrence of a connective in the well-formed formula by the occurrence of its dual connective. One of the important principles of duality is that if two well-formed formulas are truth-functionally equivalent, then their duals are also truth-functionally equivalent. The interest in duality lies in the fact that by appealing to this principle of duality, we can assert a dual equivalence (or theorem) corresponding to a given equivalence. For example, we can assert the dual equivalence

$$p \vee (p \wedge q) \Leftrightarrow p$$

when we have asserted the equivalence

$$p \wedge (p \vee q) \Leftrightarrow p .$$

Given a truth-function $f(x_1, x_2, \ldots, x_n)$ in the two-valued propositional calculus, its dual is defined to be the truth-function $\neg f(\neg x_1, \neg x_2, \ldots, \neg x_n)$ where '$\neg$' is the singulary connective 'negation'. As there is only one negation connective in the two-valued propositional calculus, the dual of a truth-function (alternately, the principal dual of a well-formed formula) can be uniquely determined. We notice that the definition of duality is such that

(i) the identity and the negation functions are self-dual;
(ii) the dual of the dual of a truth-function is the original truth-function (alternately, the principal dual of the principal dual of a well-formed formula is the same as the well-formed formula);
(iii) the semantic notions of tautology and contradiction are mutually dual.

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