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INTUITIONISTIC NEGATION

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Within Heyting's intuitionistic mathematics there are at least two distinct types of negation. The first is that which Heyting [1] (p. 18) has called "de jure" falsity. If p is a proposition then the negation of p has been proved, $\vdash \sim p$, if it has been shown that the supposition of p leads to a contradiction. That is, $\vdash p \rightarrow F$ where F is any contradiction. Intuitionistically, if p and q are propositions then $\vdash p \rightarrow q$ if a construction has been effected which together with a construction of p would constitute a construction of q. While Heyting holds that only "de jure" negation should play a part in intuitionistic mathematics [1] (p. 18), there has been a second type of negation introduced into Heyting's work which I have called "in absentia" falsity. That is $\vdash \sim p$ if it is certain that p can never be proved. This "in absentia" negation is used explicitly by Heyting in [1] (p. 116, lines 16, 17) and mentioned in [2] (pp. 239-240). In this paper I wish to show that "de jure" falsity and "in absentia" falsity lead to a contradiction in informal intuitionistic mathematics.

Consider the following definitions:

Definition 1 (vide [1], p. 115) A proposition p has been *tested* if $\vdash \sim p \vee \sim \sim p$.

Definition 2 A proposition p has been *decided* if $\vdash p \vee \sim p$.

It is well known that because of the intuitionistic interpretation of disjunction, $\vdash p \vee q$ if and only if at least one of $\vdash p$ or $\vdash q$. Consequently $q \vee \sim q$ does not possess universal intuitionistic validity so long as there are undecided mathematical problems.

Proposition 1 *A decided proposition has been tested.*
Proof: $\vdash p \rightarrow \sim \sim p$.

In a chapter on "Controversial Subjects", Heyting [1] presents some intuitionistic results of Brouwer which if interpreted classically mean that classical mathematics is contradictory.

Proposition 2 (i.e., Theorem 2, [1], p. 118) *It is contradictory, that for every real number (generator)* a, $a \neq 0$ *would imply* $a \not> 0 \vee a \not< 0$.

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