Notre Dame Journal of Formal Logic
Volume XII, Number 2, April 1971

TRUTH-VALUE SEMANTICS FOR A LOGIC OF EXISTENCE

HUGUES LEBLANC

1. Introduction. Recall the opening moves in the interpretation of a first-order language $\mathfrak{L}$: (i) items, thought of as forming a domain D, are made the values of the (bound) individual variables of $\mathfrak{L}$, (ii) a member of D is assigned to each individual constant of $\mathfrak{L}$, (iii) a (possibly empty) set of members of D is assigned to each one-place predicate constant of $\mathfrak{L}$, (iv) a (possibly empty) set of pairs of members of D is assigned to each two-place predicate constant of $\mathfrak{L}$, and so on. Löwenheim's theorem of 1915 tells us that, as regards logical truth (i.e., truth under any interpretation whatever), logical falsehood (falsehood under any interpretation whatever), and the like, all but $\aleph_0$ members of any infinite domain D may be discounted.[1] A 1959 theorem of Beth's (implicit in results of Henkin, Hasenjaeger, and others) goes one better, and tells us that, as regards logical truth, logical falsehood, and the like, all but such members of any domain D as have been assigned to the individual constants of $\mathfrak{L}$ may be discounted, *provided* $\mathfrak{L}$ *has* $\aleph_0$ *individual constants*.[2] The latter result supplies the rationale for the "substitution" interpretation of the quantifiers, according to which a universal quantification $(\forall X)A$ of $\mathfrak{L}$ is true if every replacement of X everywhere in A by an individual constant of $\mathfrak{L}$ is true, an existential one $(\exists X)A$ true if some replacement of X everywhere in A by an individual constant of $\mathfrak{L}$ is true.[3]

Like considerations apply to the first-order quantificational calculus QC. Suppose that, as in many recent presentations of QC, two different runs of letters ($\aleph_0$ letters per run) serve as individual variables: one run—for which the appellation "individual variables" is often saved—occurring only bound in the well-formed formulas (wffs) of QC, and one run—called *individual parameters*—occurring only free in them. Löwenheim's theorem tells us that, as regards validity, contravalidity, and the like, any domain whose members are made the values of the individual variables of QC may be presumed of size $\aleph_0$; Beth's that all but such members as have been assigned to the individual parameters of QC may be discounted.

Beth's theorem issues into a fresh characterization of the valid wffs of QC—as a matter of fact, into a truth-value semantics for QC that runs largely like the ordinary semantics for the sentential calculus SC. Given

Received May 15, 1969