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ON WEAK AND STRONG VALIDITY OF RULES FOR THE PROPOSITIONAL CALCULUS

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Harrop [1], [2] has distinguished between rules of inference for propositional calculi which are strongly valid and those which are only weakly valid. He gives an example of a rule which is weakly but not strongly valid with respect to a certain three valued model. Setlur [3] has shown that these notions coincide in the usual model for the classical propositional calculus. We shall show that Setlur's proof is dependent on the definition of rule given by Harrop. In particular, we generalize the notion of rule and then give some examples of rules which are weakly but not strongly valid with respect to the usual model for the classical propositional calculus.

Harrop defines a rule as a metastatement of the form $\alpha_1, \alpha_2, \ldots, \alpha_n \vdash \beta$ where $\alpha_1, \alpha_2, \ldots, \alpha_n, \beta$ are formula schemes, i.e., metalogical variables built up from variables for arbitrary formulas (vafs) and the connectives. By an application of a rule he means a statement of the form $X_1, X_2, \ldots, X_n \vdash Y$ where $X_1, X_2, \ldots, X_n, Y$ result from $\alpha_1, \alpha_2, \ldots, \alpha_n, \beta$ by a common substitution. A rule is weakly valid with respect to a certain finite model iff whenever the premises of an application of a rule are valid then the conclusion is also valid. A rule is strongly valid with respect to a certain finite model iff for any assignment of values from the model to the vafs which occur in $\alpha_1, \alpha_2, \ldots, \alpha_n, \beta$, if the premises of the rule are all designated then the conclusion β is also.

We observe that the rule $\alpha_1, \alpha_2, \ldots, \alpha_n \vdash \beta$ is strongly valid with respect to the usual model for the classical propositional calculus iff $C\alpha_1 C\alpha_2 \ldots C\alpha_n\beta$ is a tautology (where truth values are assigned to the vafs which occur in $\alpha_1, \alpha_2, \ldots, \alpha_n, \beta$).

We call $\alpha_1, \alpha_2, \ldots, \alpha_n \vdash \beta$ a *rule with restrictions* iff $\alpha_1, \alpha_2, \ldots, \alpha_n, \beta$ are formula schemes or variables for propositional letters and where there are restrictions imposed on the occurrence (or non-occurrence) of certain propositional letters in some of the $\alpha_1, \alpha_2, \ldots, \alpha_n, \beta$.

We now give some examples of rules with restrictions which are weakly but not strongly valid with respect to the usual model for the classical propositional calculus (henceforth we shall only be concerned with this model):

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