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LATTICE–THEORETICAL AND MEREOLOGICAL FORMS OF HAUBER'S LAW

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The shortest form of an important logical law of Hauber[1]:

(i) $[\alpha\beta\gamma\delta] .\because . \alpha \cap \beta = \Lambda . \gamma \cap \delta = \Lambda . \alpha \cup \beta = \gamma \cup \delta . \supset : \alpha \subset \gamma . \beta \subset \delta . \equiv . \gamma \subset \alpha . \delta \subset \beta$[2]

is provable easily in any system of logic which contains the so-called algebra of classes (or sets), as a subsystem. It is obvious that the most general form of Hauber's theorem, viz.:

(ii) $[\alpha_1\alpha_2 \ldots \alpha_n\beta_1\beta_2 \ldots \beta_n] .\because . \alpha_1 \cap \alpha_2 = \Lambda . \alpha_1 \cap \alpha_3 = \Lambda \ldots \alpha_1 \cap \alpha_n = \Lambda . \alpha_2 \cap \alpha_3 = \Lambda \ldots \alpha_2 \cap \alpha_n = \Lambda \ldots \alpha_{n-1} \cap \alpha_n = \Lambda . \beta_1 \cap \beta_2 = \Lambda . \beta_1 \cap \beta_3 = \Lambda \ldots \beta_1 \cap \beta_n = \Lambda . \beta_2 \cap \beta_3 = \Lambda \ldots \beta_2 \cap \beta_n = \Lambda \ldots \beta_{n-1} \cap \beta_n = \Lambda . \alpha_1 \cup \alpha_2 \cup \ldots \cup \alpha_n = \beta_1 \cup \beta_2 \cup \ldots \cup \beta_n . \supset : \alpha_1 \subset \beta_1 . \alpha_2 \subset \beta_2 \ldots \alpha_n \subset \beta_n . \equiv . \beta_1 \subset \alpha_1 . \beta_2 \subset \alpha_2 \ldots \beta_n \subset \alpha_n$; *where n is a natural number*: $1 < n < \infty$.[3]

can be proved at once by an application of the same mode of proof which was used in order to obtain (i). It is well known that the classical propositional calculus contains a theorem, viz.:

$\mathcal{P}$ $[pqrs] .\because . p \vee q . \equiv . r \vee s : p \supset \sim q . r \supset \sim s : \supset : p \supset r . q \supset s . \equiv . r \supset p . s \supset q$

which is analogous to (i), and that a propositional thesis corresponding to (ii) also is provable without any difficulty.

In this note it will be shown that: 1) the Boolean algebraic formulas corresponding to (i) and (ii) are provable not only in Boolean algebra which has been proved by Alves, *cf.* [1], but also in a weaker system, namely in the field of distributive lattice with Boolean zero element, and that: 2) the

1. Sometimes and especially when Hauber's law is given in its propositional form it also is called the law of closed systems, cf. [4], pp. 176-177. Hauber's original formulation of his law can be found in Hoormann's paper [2].

2. A notation used in this paper is the well-known symbolism of Peano-Russell which for the formulas belonging to lattice theory or mereology is adjusted to the requirements of these two systems respectively.

3. Cf. [3], p. 288, formula μ.

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