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GENTZEN-LIKE SYSTEMS FOR PARTIAL PROPOSITIONAL CALCULI: I

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1. *Introduction*. The existence of many decidable partial propositional calculi leads to a desire for the construction of Gentzen-like systems (with provable cut-elimination theorems) for such calculi. Two problems arise in the construction of such systems. First, one must to some extent formalize what one means by a Gentzen-like system. Second, one must find appropriate deduction rules for the systems so that the cut-elimination theorem can be proved. Here we consider the general definition and the two simpler one axiom sub-partial propositional calculi of the system of [1]. Part II of this paper will consider the other one axiom subcalculus.

2. *Classical Gentzen-like Systems*. The classical Gentzen-like systems, for example [2], might be characterized as follows.

(i) The objects manipulated are n-tuples of well-formed formulae.
(ii) Certain of these n-tuples with (easily) effectively recognizable forms are taken as axioms.
(iii) Deduction rules each take some n-tuple (or pair of n-tuples) and make (easily) effectively recognizable changes in the n-tuples. Most of these changes are permutations of the symbols of the n-tuples or the introduction of a new symbol or formula into the n-tuple.
(iv) A certain (easily) effectively recognizable subset of the n-tuples constructed from the axioms by the deduction rules is identifiable with the set of theorems of the classical theory for which the Gentzen-like system is being constructed.
(v) Often a theorem (cut-elimination) is provable on the basis of which one can effectively construct, given a n-tuple of the proper form for (iv), a finite set of possible proofs for it in such a way that one can actually decide if it is a theorem by determining if one of these possible proofs is actually a proof.

3. *Definitions*. Given a partial propositional calculus **C** with language **L** with the usual variables, constants, connectives and formation rules, let #

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