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FINITE MODEL PROPERTY FOR FIVE MODAL CALCULI IN THE NEIGHBOURHOOD OF S3

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That Lewis' system S3 is decidable was shown by Matsumoto in [9]. That it has the finite model property (f.m.p.) has been established only recently by Lemmon in [7]. First it is proved that a weaker system E3 has the f.m.p. and from this it is inferred that S3 also has the same property. There is one disadvantage to this method. It is not clear how to modify it to show that a system which is somewhat stronger (or weaker) than S3 also has the f.m.p. Given a *direct* proof this can be fairly easily done. Halldén, for example, has, in an obvious manner, extended the result from S2 to S6 (compare Theorem 5 of [10] with Theorem 13 of [5]). A similar extension from S3 to S7 is not readily available from Lemmon's treatment; and the same remark applies to weakening the result to, say, S3°.

In this paper I shall give a direct proof of the f.m.p. of S3° and extend it to the systems R3°, S3.1, S7 and S8. The system S3° is due to Sobociński [13]; R3° due to Canty [2]; S3.1, S7 and S8 due to Halldén [5]. The name "S3.1" occurs in [6]; p. 345. In §1 new axiomatizations of these systems will be given. The two important deductions of §1, those of 1.2 and 2.1, are extracted from certain considerations of Lemmon [7], both algebraic and logistical (see pp. 195-196). In §2 the f.m.p. will be established. The results of §2 are simple consequences of the axiomatizations and the author's results of [12] and thorough acquaintance with [12] is presupposed. All the terminology and notation of §2 is that of [12].

§1. AXIOMATICS. We suppose our systems to be N-K-M calculi with the usual definitions. The five systems mentioned are defined as follows:

(1) S3° = {S1°; $\mathfrak{C}\mathfrak{C}pq\mathfrak{C}MpMq$};
(2) R3° = {S3°; $CLpp$};
(3) S3.1 = {S3; $M\mathfrak{C}LpLLp$};
(4) S7 = {S3; MMp};
(5) S8 = {S3; $LMMp$}.

Now consider the following five theses:

V1 $\mathfrak{C}MKMpNMKpNpMp$;

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