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AFFINE GEOMETRY HAVING A SOLID AS PRIMITIVE

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INTRODUCTION In this dissertation we consider the problem of constructing a system of geometry devoid of such geometrical primitives as points, lines, and surfaces; and admitting as primitives only solids or "chunks". In other words can we start with some solid as a primitive term and from there add appropriate axioms to obtain a geometrical system equivalent to a well known geometrical system such as Euclidean geometry or affine geometry?

Part of the answer was given by A. Tarski (in [8]) who solved the problem for ordinary Euclidean geometry. His solution was as follows:

(1) He begins with an axiomatization of the relation "*A* is part of *B*" (This deductive system is due to Leśniewski (see [5]) and is called Mereology (see Appendix A)).

(2) He adds to Mereology the primitive term sphere and uses only the part relation and sphere to define the notion point-class and the notion of equidistance among point-classes.

(3) Then he adds the ordinary axioms of Euclidean geometry (based on point and equidistance as primitive—see M. Pieri [6]) replacing the primitive terms by the defined terms above.

(4) Finally if we interpret sphere as open ball in ordinary Euclidean geometry there is a bijective correspondence between point-classes and points; and Tarski's system is equivalent to Euclidean geometry.

We shall consider a generalization of Tarski's result in which we solve the above problem for the class of affine geometries which are equivalent to finite dimensional vector spaces over an ordered field.

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