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COMPLETENESS OF THE GENERALIZED PROPOSITIONAL CALCULUS

ALEXANDER ABIAN

By a *Generalized Propositional Calculus* we mean the Classical Propositional Calculus with *any number* (countable or uncountable) of atomic formulas (propositions) $p, q, r, s, \ldots$.

In this paper we prove that the Completeness theorem for the Generalized Propositional Calculus, i.e., the statement: *"A formula of the Generalized Propositional Calculus is a theorem if and only if it is a tautology"*, is equivalent to the Prime Ideal theorem for Boolean rings.

By a Boolean ring we mean a Boolean ring with more than one element and by the Prime Ideal theorem for Boolean rings we mean any of the following pairwise equivalent statements.

(1) *Every Boolean ring has a proper prime ideal.*

(2) *For every element P^* of a Boolean ring Γ such that P^* is not the multiplicative unit of Γ there exists a nontrivial homomorphism from Γ onto the two-element Boolean ring $\{0, 1\}$ which maps P^* into* 0.

(3) *Every Boolean ring with a multiplicative unit has a proper prime ideal.*

For the Generalized Propositional Calculus we choose as the primitive logical connectives the *negation* denoted by "$\sim$" and the *disjunction* denoted by "v". These primitive connectives together with the grouping symbols i.e., the parentheses "(" and ")" are used in the usual manner for forming *formulas* (propositions).

The logical connectives $\wedge, \oplus, \rightarrow$ and $\leftrightarrow$ are introduced as abbreviations given by:

$P \wedge Q$	for	$\sim(\sim P \vee \sim Q)$
$P \oplus Q$	for	$(P \wedge \sim Q) \vee (\sim P \wedge Q)$
$P \rightarrow Q$	for	$\sim P \vee Q$
$P \leftrightarrow Q$	for	$(P \rightarrow Q) \wedge (Q \rightarrow P)$

where P and Q are metalinguistic symbols (formula schemes) standing for formulas.

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