LOGICAL AND HISTORICAL REMARKS ON SACCHERI'S GEOMETRY

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Resumen

El autor se ocupa de la geometría contenida en el libro *Euclides ab omni naevo vindicatus*, 1733, de Girolamo Saccheri. Después de plantear el problema en §1, el autor analiza en §2 los paralogismos cometidos por Saccheri, cuando éste rechaza las geometrías del ángulo obtuso (elíptica) y del ángulo agudo (hiperbólica). En §3 expone el método de Saccheri, basado principalmente en la ley de Clavius, $(\sim p \supset p) \supset p$, y muestra su influencia y sus profundas consecuencias en la evolución del concepto de geometría en Saccheri, Lambert, Taurinus y Gauss. Finalmente el autor en §4 intenta hacer comprensibles los motivos que impulsaron a Saccheri a cometer sus paralogismos, siendo el principal motivo la situación histórica y filosófica contemporánea.

§1. *Introduction: Saccheri's Geometry*. In this paper*, we shall be concerned with the treatise *Euclides ab omni naevo vindicatus* (1733) by Girolamo Saccheri[1][17], which is a text on *real elementary plane geometry*, and as a matter of fact is the first one without assuming Euclid's fifth postulate. Therefore we shall consider only *plane geometry*, that is an aggregate or universe of points and (straight) lines, that are related by relations of incidence and order or any other equivalent. Also we consider only *elementary* geometry, that is Riemannian geometry of constant curvature or in a more historical way geometry in which there is a very specific relation of congruence among figures. Finally we consider only *real* geometry, that is assuming the postulates of continuity of Hilbert or the equivalent one of Dedekind.

We resort to the axiomatic method for a precise formulation. The axioms of elliptic, Euclidean and hyperbolic (real, elementary) plane geometries are well established and known, and we need not list them here.

Received July 22, 1969