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AXIOMATIC QUASI-NATURAL DEDUCTION

JOHN R. GREGG

I. Two distinct methods are available for constructing proofs in quantification theory: deduction from axioms and "natural" deduction from premises. The first method has the formal advantage that each line of a proof is valid; but proofs turn less upon strategy than upon brute ingenuity, and often are hard to come by. The second method, generally speaking, is the more convenient and perspicuous, proofs turning more upon strategy and somewhat less upon insight; but these niceties are paid for, sometimes dearly, in more or less artificial restrictions upon proof format and upon the use of rules of inference.

The burden of this paper is to describe a new axiom system G; to show that adoption of some simple, transparent and purely typographical conventions yields a method of proof—called *axiomatic quasi-natural deduction*—in which are combined all advantages of deduction from axioms with those of deduction from premises; and to show that the system is both complete and sound in the sense that all and only valid quantificational formulae are among its theorems.

II. The primitives of G are the constant (inconsistency) 'f', sentence letters ('p', 'q', 'r', 's' and their subscripted variants), n-place predicate letters ('F^n', 'G^n', 'H^n' and their subscripted variants), variables ('w', 'x', 'y', 'z' and their subscripted variants), parentheses, the conditional sign '$\supset$' and the universal quantifier sign '$\forall$'.

The formulae of G are all and only expressions identified recursively by these rules: (i) 'f' is a formula, (ii) a sentence letter is a formula, (iii) an n-place predicate letter followed by a string of n variables is a formula, (iv) any result of putting a formula for 'p' and one for 'q' in '$(p \supset q)$' is a formula, (v) any result of putting a variable for 'x' and a formula for 'p' in '$\forall xp$' is a formula.

Henceforth, 'P', 'Q', 'R' and their subscripted variants will refer ambiguously to formulae, 'X' and 'Y' to variables. Thus, $(P \supset Q)$ is the conditional whose antecedent is P and whose consequent is Q, $\forall XP$ is the universal quantification of P with respect to X, and so on.

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