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# THE NON-EXISTENCE OF A CERTAIN COMBINATORIAL DESIGN ON AN INFINITE SET*

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In [1] the notion of a combinatorial design on an infinite set $M$ was based on a covering relation of the following kind.

*Definition 1. Let $F$ and $G$ be two families of subsets of $M$ and let $p$ be a non-zero cardinal number. $G$ is said to be a $p$-Steiner cover of $F$ if and only if every member of $F$ is contained (as a subset) in exactly $p$ members of the family $G$.*

We showed in [1], roughly speaking, that a rather large class of families $F$ possess $p$-Steiner covers of a specified nature. To be more exact, we introduce the following additional definitions.

*Definition 2. Let $k$ be a non-zero cardinal number such that $k \leq \overline{\overline{M}}$. A family $F$ of subsets of $M$ is called a $k$-tuple family of $M$ if and only if* i) *if $x, y \in F$ such that $x \neq y$ then $x \not\subset y$,* ii) *if $x \in F$ then $\overline{\overline{x}} = k$ and* iii) $\overline{\overline{F}} \leq \overline{\overline{M}}$.

In terms of Definitions 1 and 2 we can state the main result of [1] as

*Theorem 3.*[1] *Let $v$, $k$, $n$ and $p$ be non-zero cardinal numbers such that* i) *$v$ is non-finite,* ii) *$k < n < v$, and* iii) *$p \leq v$. Then if $M$ is a set of cardinality $v$ every $k$-tuple family $F$ of $M$ possesses a $p$-Steiner cover $G$ such that every member $y \in G$ is a subset of $M$ of cardinality $n$.*

A natural question arises as to whether Theorem 3 would be true if restriction iii) of Definition 2 were removed. The present paper's aim is to show this restriction is necessary.

All results achieved in the present paper are formalizable within Zermelo-Fraenkel set theory with the axiom of choice. For the most part the notation will be standard. If $x$ is a set, $\overline{\overline{x}}$ will represent the cardinal number of $x$. Moreover, if $n$ is any cardinal number then $[x]^n = \{y \subset x: \overline{\overline{y}} = n\}$.[2] The expression "$x \subset y$" means "$x$ is a subset of $y$" improper inclusion not being excluded. If $\alpha$ is an ordinal $\omega_\alpha$ is the smallest ordinal number whose cardinality is $\aleph_\alpha$. As usual we write $\omega$ for $\omega_0$.

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