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ON THE CALCULUS MCC

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Charles Parsons, in [2], has very nicely tidied up a topic on which I had some correspondence with the late E. J. Lemmon early in 1957, and it may now be of some interest to bring this material into the light of day.

The calculus which Parsons calls MC^+, i.e. Johannson's minimal calculus + *ApNp*, is equivalent to the calculus which H. B. Curry in [1] calls **LD**. When I first learnt of this calculus it occurred to me that one might obtain another calculus between the minimal and the classical by leaving negation as in minimal but making the implicational basis fully classical. This gives the calculus which Parsons calls **MCC**. All that I observed about it was that it was weaker than the full calculus but stronger than Curry's, and I proved it not to be contained in Curry's by the matrix

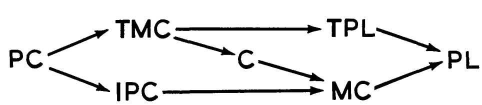
C	1	2	3	*N*
*1	1	2	3	2
2	1	1	3	1
3	1	1	1	1

A	1	2	3
1	1	1	1
2	1	2	2
3	1	2	3

K	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

(N = (1,1,1) would also do). But Lemmon noticed a good deal more, and I cannot do better than quote the relevant portion of a letter from him of January 18, 1957:

"Let **PL** be positive logic (in *C*, *K*, *A*); **MC** = minimal calculus (*C*, *K*, *A*, 0 or *C*, *K*, *A*, *N*); **TPL** = classical positive logic (in *C*, *K*, *A* with *CCCpqpp*); **C** = Curry's system (**MC** in *C*, *K*, *A*, 0 or *C*, *K*, *A*, *N* + *CCNppp* or *ApNp*); **TMC** = the system you describe (**MC** + *CCCpqpp*, or **TPL** + 0); **IPC** = the full intuitionistic propositional calculus; **PC** = full classical propositional calculus. Then, as you say:



(where relationships are not given, independence obtains). Then these systems are described by the following equations, using *C*, *K*, *A* and 0

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