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ON SOME OPEN QUESTIONS OF B. SOBOCIŃSKI

G. F. SCHUMM

In Sobociński's [2] and [3] several questions are left open, among them

(1) Is K1.1 a proper extension of K1?
(2) Is K2.1 a proper extension of K2?
(3) Is K3.1 a proper extension of K3?
(4) Does there exist a system intermediate between S4.4 and S5?

With the aid of the matrices

$\mathfrak{M}$

C	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*N*
**1*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*8*
2	*1*	*1*	*3*	*3*	*5*	*5*	*7*	*7*	*7*
3	*1*	*2*	*1*	*2*	*5*	*6*	*5*	*6*	*6*
4	*1*	*1*	*1*	*1*	*5*	*5*	*5*	*5*	*5*
5	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*	*4*
6	*1*	*1*	*3*	*3*	*1*	*1*	*3*	*3*	*3*
7	*1*	*2*	*1*	*2*	*1*	*2*	*1*	*2*	*2*
8	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*

$\mathfrak{M}'$

p	*M*	*L*
**1*	*1*	*1*
2	*1*	*4*
3	*1*	*4*
4	*1*	*4*
5	*5*	*8*
6	*5*	*8*
7	*5*	*8*
8	*8*	*8*

$\mathfrak{M}''$

p	*M*	*L*
**1*	*1*	*1*
2	*1*	*8*
3	*1*	*8*
4	*4*	*8*
5	*1*	*5*
6	*1*	*8*
7	*1*	*8*
8	*8*	*8*

all four questions are here answered in the affirmative, a familiarity with [2] and [3] being presupposed.

Ad (1)-(3). Matrices $\mathfrak{M}$ and $\mathfrak{M}'$ verify K1, K2 and K3 but falsify *CLCLCpLppp* for *p/3*: *CLCLC3L333* = *CLCLC3433* = *CLC433* = *C13* = *3*.

Ad (4). We exhibit such a system and show it to be Halldén-incomplete in the sense of [1], i.e., to contain wffs α and β having one variable each and no variable in common and such that $A\alpha\beta$, but neither α nor β, is a thesis.

Consider the system S4.7 obtained by adding *ALCMpLMpLCLMqMLq* as an axiom to S4.4. Matrices $\mathfrak{M}1$ and $\mathfrak{M}2$ of [2] verify S4.7 but falsify the S5 thesis *LCMpLMp*, while $\mathfrak{M}1$ and $\mathfrak{M}3$ verify S4.7 but falsify *LCLMqMLq*. S4.7 is thus a Halldén-incomplete extension of S4.4 and is properly

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