Notre Dame Journal of Formal Logic
Volume X, Number 3, July 1969

COMPLETE MODALIZATION IN S4.4 AND S4.0.4

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In [1] bases for S4 and S4.2 were offered which didn't require axioms beyond those of the **PC**; in [2] a version of the deduction theorem was presented giving a unified treatment of this metatheorem for S4, S4.2, and S5. The central idea in these papers was the notion of "complete modalization"; the three different concepts of complete modalization presented serve as characterizations of these three distinct systems.

Two other systems which lend themselves to the treatment of those papers are S4.4 [4] and S4.0.4 [3]. It is possible to find modifications of the notion of complete modalization chracteristic of each of these systems. We recall first of all that S4.4 and S4.0.4 result from the addition, respectively, of $CpCMLpLp$ and $CpCLMLpLp$ to S4 (we assume, of course, that the added axiom comes under the sway of a rule to infer $L\varphi$ from any theorem φ; if S4 is thought of as being in the original Lewis formulation rather than a Lemmon-type [5] base, we would add versions of these extra axioms with strict, rather than material, implication as the main connective).

We now state the rules of [1] for the introduction of L in antecedent and consequent of an implication:

RL1: $C\alpha\beta \rightarrow \vdash CL\alpha\beta$
RL2: $C\alpha\beta \rightarrow \vdash C\alpha L\beta$, *provided* α *is completely modalized.*

Complete modalization in S4 may be recursively defined as follows:

(a) If φ is an S4 theorem, φ is completely modalized in S4
(b) $L\varphi$ is completely modalized in S4.
(c) If φ and ψ are both completely modalized in S4, so too is $K\varphi\psi$.

We now extend the definition to S4.4 and S4.0.4:

(d) If φ is completely modalized in S4, it is completely modalized in both S4.4 and S4.0.4.
(e) $K\varphi ML\varphi$ is completely modalized in S4.4
(e') $K\varphi LML\varphi$ is completely modalized in S4.0.4

Received February 15, 1968