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MODAL SYSTEMS IN WHICH NECESSITY IS "FACTORABLE"

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We will say that necessity is "factorable" in a modal system S if there are modal functions $X_1p, \ldots, X_np$ — L itself being none of the X_i — such that in S the conjunction $KX_1pKX_2p \ldots X_np$ is equivalent to Lp. For the systems discussed in this paper, n in the above formulas will be 2 and X_1p will be simply p. An obvious example of a system in which necessity is factorable is the system S4.4, which contains as a thesis

(1) $$EKpMLpLp.$$

We shall redirect our attention to S4.4 later on in this paper.

1. S *images in the* S° *systems*. We shall now show that by considering the operator usually read as "necessity" in the systems S1°-S4° to be a factor of necessity rather than necessity itself, we may find in each of these systems an image of its respective (without the '°') ordinary Lewis-modal system. As bases for S1°-S4°, we may use the *C-N-L* formulations of [1]; for our present purposes, however, let us employ for these systems the letter Q in place of L, and reserve L for the necessity operator in the "images" we will discover in S1°-S4°. In all of these systems, then, we will define L and M as follows:

Df. L: $L\varphi$ for $K\varphi Q\varphi$
Df. M: $M\varphi$ for $ANQN\varphi\varphi$

Axioms and rules for the systems will be drawn from the following stock, as in [1], with Q read for L:

J1a. $CQCpCqrQCQpCQqQr$
J1b. $CQCpqCQpQq$
J2. $CKQCpqQCqrQCpr$
Ja. If $\vdash\varphi$, then $\vdash Q\varphi$.
Jb. *If* φ *is an axiom or* **PC** *theorem,* $\vdash Q\varphi$.
Jc. *If* $\vdash QC\varphi\psi$, *then* $\vdash QCQ\varphi Q\psi$.
Jd. *If* $\vdash QC\varphi\psi$ *and* $\vdash QC\varphi\psi$, *then* $\vdash QCQ\varphi Q\psi$
Je. *If* $\vdash Q\varphi$, *then* $\vdash\varphi$.

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