Notre Dame Journal of Formal Logic
Volume X, Number 1, January 1969

THEORY OF PROCEDURES
I. SIMPLE CONDITIONALS

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Introduction. The aim of this paper is to set up the beginnings of a logical theory of procedures, one of the major uses of which will be as a theory of computation. Much of the initial portion of the paper consists of a generalization and re-setting of the theory developed in McCarthy [6]. (For the translation of McCarthy's theory into practice, see McCarthy et al. [7].) In [9], Thiele has developed a similar theory, oriented especially toward algorithmic languages such as are used in digital computer programming. It is not to be thought that a theory of *computability,* such as is provided by Turing machines or Markov algorithms[1] is in any way a theory of computation or a general theory of procedures. In fact the latter is used repeatedly in an informal fashion whenever one constructs Turing machines or Markov algorithms.

In section 1, we describe informally the nature and structure of procedures, giving examples of procedures and of the way in which they may be diagrammed and symbolized. The morphology of a formal theory for procedures is set out in section 2, and in section 3 and section 4 interpretations are given for the various parts of this formalism. In section 5, a theorem on the eliminability of propositional operators within the theory is proved. Then in section 6 we prove a normal form theorem, which yields a sufficient condition for complete axiomatizations of procedure theory at this first level, where only simple conditionals occur.

1. *The Structure of Procedures.* The treatment of procedures begins from a different point from the theory of Turing machines or Markov algorithms. In these theories, a *space* is assumed at the very start; for Turing machines the space is the tape which the machine uses for reading, shifting or printing, and for Markov algorithms the space is the alphabet over which the algorithm operates. A Turing machine is assessed in terms of its effect on the tape, in particular the 'number' which it writes on the tape as its final output, and Markov algorithms are assessed in terms of their effect on words drawn from their alphabet, in particular their terminal production. In procedure theory we can assess procedures purely

Received July 20, 1967