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CONCERNING SOME PROPOSALS FOR QUANTUM LOGIC

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The suggestion made in 1936 by Birkhoff and von Neumann and discussed by Birkhoff in [1], pp. 156-163 that a propositional algebra appropriate to quantum theory should have the structure of an orthocomplemented lattice has been widely discussed. More recently Kochen and Specker [2], pp. 177-189 have presented the idea of a partial Boolean algebra. We will call a partial Boolean algebra B complete if each Boolean subalgebra of B is complete.

It is the purpose of this note to point out that a complete partial Boolean algebra has an extension to an orthocomplemented lattice L, and thus may be considered as such a lattice in which the join and meet of a pair x, y of elements of L is of logical significance if and only if each of x and y belongs to the same Boolean subalgebra of L, i.e., x, y is a commeasurable pair in the sense of [2]. Otherwise $x \smile y$ and $x \frown y$ are meaningless for quantum logic although existing in the lattice-theoretic sense. Perhaps this observation will help to clarify one of the problems frequently mentioned (e.g. in [3], p. 369) which is involved in the structure of a logic for quantum theory.

Instead of using the definition of a partial Boolean algebra, it will be more convenient for our purpose to have recourse to the properties of a model thereof, since [2] p. 184, every partial Boolean algebra is isomorphic to some case of the model. Hence the statement that B is a partial Boolean algebra will mean that B is a list $(M, \vee, \neg, 0, 1)$ and I is a set such that if $i \varepsilon I$, then B_i is a Boolean algebra $(M_i, \vee_i, \neg_i, 0, 1)$ and

(i) $M = \bigcup_{i \varepsilon I} M_i$;

(ii) if $h, i, j \varepsilon I$, $a, b \varepsilon M_h$, $a, c \varepsilon M_i$ and $b, c \varepsilon M_j$, then there is a $k \varepsilon I$ such that $a, b, c \varepsilon M_k$;

(iii) if $i, j \varepsilon I$, there is a $k \varepsilon I$ such that $M_i \cap M_j = M_k$;

(iv) if $a, b \varepsilon M$, then $a \vee b \varepsilon M$ if and only if there is an $i \varepsilon I$ such that $a, b \varepsilon M_i$, whence $a \vee b = a \vee_i b$;

(v) if $a \varepsilon M_i$, then $\neg a = \neg_i a$.

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