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THE NUMERICAL EPSILON

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In this paper* Leśniewski's system of ontology extended by an axiom of infinity is used to derive Peano's arithmetic. Section 1 gives the main theses of this derivation which parallels the work of [6]. Using the numerical epsilon, defined in section 2, Peano's arithmetic is given a characteristically ontological model in section 3. Thus, the paper provides, for Peano's arithmetic, the two ways of treating logical concepts in ontology, the one, protothetical (section 1), the other, ontological (section 3).

1. *Numerals as predicates* The following proposition, in which the epsilon is primitive and is a proposition forming functor for two name arguments, is taken as the single axiom of ontology and is understood to be added to some given development of protothetic.

$$[Ab] \therefore A \varepsilon b . \equiv : [\exists C] . C \varepsilon A : [C] : C \varepsilon A . \supset . C \varepsilon b : [CD] : C \varepsilon A . D \varepsilon A . \supset . C \varepsilon D$$

There is no rule which determines the style of letters to be used for variables, but throughout the paper capital Latin letters will be used for proper name variables and lower case Latin letters for general name variables; Greek letters will be employed for variables of higher semantical categories. Two types of definition, with the usual restrictions for bound and free variables, are allowable in ontology; ontological definitions which have the form:

$$[Aabc \ldots] : [\exists b] . A \varepsilon b . \Phi(Aabc \ldots) . \equiv . A \varepsilon \tau \langle abc \ldots \rangle$$

and protothetical definitions which are of the form:

$$[abc \ldots] : \Phi(abc \ldots) . \equiv . \tau(abc \ldots)$$

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