Notre Dame Journal of Formal Logic
Volume X, Number 1, January 1969

REMARKS ON CLASSIFICATION OF THEORIES BY THEIR COMPLETE EXTENSIONS[1]

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As far back as in 1928 A. Tarski introduced the concept of *degree of completeness* of a theory, [2]. Subsequently he distinguished between cardinal and ordinal degree of completeness, [3], and refined the former to the so-called *characteristic pair* of a theory, [4]. In 1936, 1937, A. Mostowski made a thorough study of the relationship between the characteristic numbers and the *structural type* of a theory, [1]. Closely related with the work of Tarski and Mostowski is A. Lindenbaum, who recognized, independently, the importance of the concept of structural type and whose theorem, stating that every consistent theory has a consistent and complete extension, is basic to the studies involved.

This paper intends to introduce the student of Logic to the metamathematics mentioned above, but by a method different from those used by the original authors. Tarski's contributions to the subject matter arise, more or less as corollaries, within a mighty axiomatic framework intended to cover the whole of the theory of theories, too heavy, it seems, for a student who as a start wants to study, say, classification of theories. Mostowski's paper on the matter requires more knowledge of topology than the average logician, who is not a mathematician, possesses.

The method employed in this paper could be called "naive". However, starting "from scratch", with structures established in the form of trees, it arrives at the desired results in a simple and rather self-contained way (only some fragmentary acquaintance with cardinal and ordinal arithmetic is required). Results coinciding with results of Lindenbaum, Mostowski, Tarski are mentioned as theorems. Side-results, due to the particular method, are mentioned as remarks. The terminology, in so far as it is new, is intended *ad hoc*, i.e., to promote readability of this paper.

§1. Introductory Remarks. Our considerations are confined to a first-order language with identity containing no other than non-logical constants

Received July 5, 1968