Notre Dame Journal of Formal Logic
Volume IX, Number 4, October 1968

DEFINITIONAL BOOLEAN CALCULI

RICHARD COLE

It is often said that the truths of the propositional calculus follow (informally, to be sure) just from the meanings of the connectives and negation. Yet, formal developments of propositional calculi do not perspicuously reveal such a relation; an axiomatization may be said to "contain" the properties of the operators therein embedded but they do not do so in any clear way. What we should like is a formal development of propositional calculi with intuitive rules of inference and with axioms which are, or are like, truth tables.

In this paper I shall develop a class of formal systems, to be called "definitional boolean systems", which can, so to speak, be understood to reveal that the theorems of a propositional calculus do follow from (what amount to) definitions of the appropriate operators. Using "0" and "1" for the truth values "false" and "true" I shall, for example, represent the sense of negation by:

$$N0 = 1$$

and

$$N1 = 0.$$

We may associate these formulas with the proposition: if the truth value of a proposition, X, is false (true) then the truth value of the negation of X is true (false).

Following through for certain other operators, we represent the meanings of:

$N0$. $N0 = 1$	$A00$. $A00 = 0$	$K00$. $K00 = 0$	$C00$. $C00 = 1$	$D00$. $D00 = 1$
$N1$. $N1 = 0$	$A01$. $A01 = 1$	$K01$. $K01 = 0$	$C01$. $C01 = 1$	$D01$. $D01 = 1$
	$A10$. $A10 = 1$	$K10$. $K10 = 0$	$C10$. $C10 = 0$	$D10$. $D10 = 1$
	$A11$. $A11 = 1$	$K11$. $K11 = 1$	$C11$. $C11 = 1$	$D11$. $D11 = 0$

For convenience, I have named these formulas for the expressions to the left of the identity sign. I shall call these sets the **N** set, **A** set, **K** set, **C** set, and **D** set.

Received November 13, 1967