Notre Dame Journal of Formal Logic
Volume IX, Number 4, October 1968

THE PROPOSITIONAL CALCULUS MC AND ITS MODAL ANALOG

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In [5], Łukasiewicz sets down a system for which the matrix

p	Np
0	1
$\frac{1}{2}$	1
1	0

C	0	$\frac{1}{2}$	1
0	0	$\frac{1}{2}$	1
$\frac{1}{2}$	0	0	1
1	0	0	0

K	0	$\frac{1}{2}$	1
0	0	$\frac{1}{2}$	1
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
1	1	1	1

A	0	$\frac{1}{2}$	1
0	0	0	0
$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
1	0	$\frac{1}{2}$	1

(with 0 as designated value) is characteristic. This system is formed by adding to the intuitionist propositional calculus (IC) the axiom

$$CCNpqCCCqpqq \qquad (1);$$

he notes that Apq may be defined in this system by the formula

$$KCCpqqCCqpp \qquad (2).$$

This definition is, of course, "characteristic" of Dummett's system LC [1] in the sense that its addition to IC yields LC. In the present section of this paper, we shall propose a definition of Apq "stronger" than that above, and will show that it is characteristic of a system—which we call MC—equivalent to that of Łukasiewicz [5]. In the latter part of this paper we shall investigate the Lewis-modal system analogous to MC.

We shall call MC the system formulable by adding to IC the definition

$$Apq \text{ for } KCNpqCCqpp \qquad (3).$$

Alternate formulations are available; if we add to IC the axiom

$$ACpqCNNqp \qquad (4)$$

or

$$ACpqCCNqpp \qquad (5)$$

or, finally

$$ANpAqCqp \qquad (6)$$

the result will be MC. For the moment, let us call IC + (4) MC', and

Received July 24, 1967