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A STANDARDIZATION THEOREM FOR STRONG REDUCTION

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In a previous paper [2], the writer introduced a modified definition of strong reduction in combinatory logic [1a]. This paper shows that with such a strong reduction there is associated a standard reduction [3a]. All reductions are considered according to the modified definition. This allows some essential simplifications in proofs, especially of Theorem 1.

Definition of a Standard Reduction.

A strong reduction is called standard if it satisfies the following conditions:

i) The Type III steps are made last and are performed from right to left.
ii) Among the Type I and II reductions the redexes are to be contracted in the order (from left to right) of the combinators appearing at their heads. However, the redex contracted need not be of maximal extent.

The condition that Type III steps be contracted from right to left is automatically satisfied in the case that they overlap; and in case they do not, the order is irrelevant.

In the context of standard reductions, steps of Type IIc, IId, and IIf have the effect of freezing certain combinators, since they introduce expressions which can only be reduced further by Type III steps. We shall refer to combinators as being frozen without resorting to the mechanism of these steps. A combinator not frozen is called *free*.

Since the transformation of a modified strong reduction into a strong reduction in the original sense involves introduction of Type III steps ahead of Type I and II steps, a standard reduction in the present sense need not be standard in the sense of the original definition.

Two Lemmas. Two lemmas were introduced in [2] and will be used here. They are:

Lemma 1. *If* $X = [x]\mathfrak{X}$, *then* $\lambda x.Xx \succ\!\!- \lambda x.\mathfrak{X}$ *by Type I steps only. In*

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