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SYNTACTICALLY FREE, SEMANTICALLY BOUND (A NOTE ON VARIABLES)

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> *Apparent variables*. The symbol "$(x).\phi x$" denotes one definite proposition, and there is no distinction in meaning between "$(x).\phi x$" and "$(y).\phi y$" when they occur in the same context.
> *Principia Mathematica*, Introduction, Ch. I.

The old distinction between an *apparent variable* and a *real* one was never too clearly drawn. Passages from *Principia Mathematica* and earlier logic treatises suggest, though, that an individual variable X *apparently* occurs in a formula A if X occurs in A just for form, i.e., if X can be replaced *salvo sensu* in A by some individual variable foreign to A, and that X *really* occurs in A if X does not *apparently* occur in A. Thus, 'x' *apparently* occurs in Russell's '$(\forall x) f(x)$' (= '$(x).\phi x$'), since 'x' can be replaced *salvo sensu* in '$(\forall x) f(x)$' by 'y', whereas 'x' *really* occurs in '$f(x)$'. The distinction between a bound variable and a free one, which eventually displaced that between an apparent variable and a real one, does not match it, all assertions to the contrary notwithstanding. An individual variable may—by current standards—occur free in a given formula, and yet not *really* occur therein by the above critierion. 'x', for example, though it occurs free in '$(\forall x) f(x)$ & $f(x)$', does not *really* occur in '$(\forall x) f(x)$ & $f(x)$', since it can be replaced *salvo sensu* in '$(\forall x) f(x)$ & $f(x)$' by any one of 'y', 'z', 'x'', 'y'', 'z'', and so on, or—as we prefer to put it—since '$(\forall x) f(x)$ & $f(x)$' is semantically equivalent to any one of '$(\forall y) f(y)$ & $f(y)$', '$(\forall z) f(z)$ & $f(z)$', and so on.[1]

Because of this discrepancy we would urge that an individual variable X, when it occurs bound (free) in a formula A by current standards, be said to occur *syntactically bound* (*syntactically free*) in A, and that a fresh distinction be introduced according to which: (i) X is said to occur *semantically bound* in A if A is semantically equivalent to any (hence, to

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