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A SEMANTICAL PROOF OF THE UNDECIDABILITY OF THE MONADIC INTUITIONISTIC PREDICATE CALCULUS OF THE FIRST ORDER

JEKERI OKEE

A constructive proof of the undecidability of the monadic intuitionistic predicate calculus of the first order was given in [3]. We shall denote this calculus with "**MIP**". The aim of this article is to give a proof of the undecidability of the **MIP**, using Kripke semantics. We shall show that the class of formulae of the **MIP** which contain only two predicate variables is undecidable. We shall denote this class by $\mathcal{L}$. It is well known that in the classical two-valued predicate calculus of the first order, the class of formulae which contain only one binary predicate variable is undecidable. We shall denote this class by $\mathcal{M}$. Let $\mathcal{K}$ be the class of all formulae of the intuitionistic predicate calculus of the first order, then we can obtain any formula $H \epsilon \mathbf{M}$ from $\mathcal{K}$, by simply interpreting classically every propositional functor and every quantifier occurring in H. Since $\mathcal{M}$ is undecidable, it follows that the class $\mathcal{K}'$ of all formulae of $\mathcal{K}$ with only one binary predicate variable is also undecidable. To prove the undecidability of $\mathcal{L}$ we shall assign, to every closed formula $H \epsilon \mathcal{K}'$, a formula $H^* \epsilon \mathcal{L}$ such that H is valid in $\mathcal{K}$ if and only if H^* is valid in **MIP**, then since $\mathcal{K}'$ is undecidable, it follows that $\mathcal{L}$ is also undecidable. The details of Kripke semantics will be assumed (see [1]).

The following definitions are given in [1], p. 94, namely:

(a) $\phi(A \wedge B, H) = \mathbf{T}$ iff $\phi(A, H) = (B, H) = \mathbf{T}$, otherwise $\phi(A \wedge B, H) = \mathbf{F}$,
(b) $\phi(A \vee B, H) = \mathbf{T}$ iff $\phi(A, H) = \mathbf{T}$ or $\phi(B, H) = \mathbf{T}$, otherwise $\phi(A \vee B, H) = \mathbf{F}$,
(c) $\phi(A \rightarrow B, H) = \mathbf{T}$ iff for all $H' \epsilon \mathcal{K}$ such that $H R H'$, $\phi(A, H) = \mathbf{F}$ or $\phi(B, H) = \mathbf{T}$, otherwise $\phi(A \rightarrow B) = \mathbf{F}$,
(d) $\phi(\sim A, H) = \mathbf{T}$ iff for all $H' \epsilon \mathcal{K}$ such that HRH', $\phi(A, H') = \mathbf{F}$, otherwise, $\phi(\sim A, H) = \mathbf{F}$.

In addition to the above definitions we shall add the following:

(e) $\phi(\sim\sim A, H) = \mathbf{T}$ iff there exist an $H' \epsilon \mathcal{K}$ such that $\phi(A, H') = \mathbf{T}$, otherwise, $\phi(\sim\sim A, H) = \mathbf{F}$.

The definition (e) is clearly consistent with the definitions (a)-(d).

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