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ALGEBRAIC LOGIC WITH GENERALIZED QUANTIFIERS

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1 *Introduction* The notion of languages with generalized quantifiers was introduced by A. Mostowski in [5]. Recently, this subject has attracted a great deal of attention and is currently undergoing a rapid development. In particular, the study of logic with the quantifier "there exist uncountably many" has become an important part of current investigations in model theory and set theory.

The object of this note is to describe the algebraic logic for calculi with generalized quantifiers. It is shown that the algebraic version of generalized quantifiers is a perfectly natural generalization of the usual notion of quantifiers in cylindric and polyadic algebras, and occurs naturally in Boolean algebras. We investigate the algebra of the structures which arise when generalized quantifiers are added to cylindric algebras, and characterize those cylindric algebras which admit generalized quantifiers. Finally, we give a few applications to the logic $\mathsf{L}(\mathsf{Q}_1)$. Our notation and terminology is that of Henkin, Monk, and Tarski [3], except that we will say "quantifier" instead of "cylindrification".

2 *Algebraic Formulation of Generalized Quantifiers* Looking at the various extensions of quantification which have recently been studied (for example [1], [4], [5], [6]), a clear notion of generalized quantifiers is seen to emerge. Algebraically, this notion may be formulated in the following terms:

2.1 Definition Let A be a Boolean algebra. By a *generalized quantifier* on A we mean a function $\mathsf{q}: A \rightarrow A$ having the following properties:

Q1 $\mathsf{q}(x + y) = \mathsf{q}x + \mathsf{q}y$
Q2 $\mathsf{q}(x \cdot \mathsf{q}y) = \mathsf{q}x \cdot \mathsf{q}y$
Q3 $\mathsf{q}0 = 0$
Q4 $\mathsf{q}1 = 1$.

One immediately observes that quantifiers in the usual sense satisfy Q1-Q4. However, they also satisfy the inequality $x \leq \mathsf{q}x$, which does not hold for any other generalized quantifiers.

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