NICE IMPLICATIONAL AXIOMS

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It might seem unlikely at this date that a new and interesting three-axiom set for classical implication would be found. However I do not remember in the literature the set $\{1, 2, 3\}$ below. In number of axioms and basic implicational structure it is identical with Tarski's $\{1, 14, 7\}$ which in some sense strengthens 2 and weakens 3; the variable occurrences are $7\,p$, $4\,q$, $4\,r$, against Tarski's $6\,p$, $4\,q$, $5\,r$. The conspicuous merit of $\{1, 2, 3\}$ is the ease with which all the most famous and commonly named propositions can be developed; we have a minimum of material which is of merely local or contextual necessity and interest. For a discussion of axiomatics I know of no other set which assembles so much needed material in such short order. Witness the following:

1. $CpCqp$ (Simp)
2. $CCqrCCpqCpr$ (Weak Syll)
3. $CCCpqrCCrpp$ (Roll)
D21 = 4. $CCqrCqCpr$ (A Fortiori)
DD243 = 5. $CCCpqrCCrpCsp$ (Łukasiewicz)
D23 = 6. $CCsCCpqrCsCCrpp$
D63 = 7. $CCCpqrCCprr$ (Tarski)
D61 = 8. $CpCCpqq$ (Aff or Pon)
DD228 = 9. $CqCCpCqrCpr$ (Comm-Comm)
DD999 = 10. $CCpCqrCqCpr$ (Comm)
DD10.1.n₁ = 11. Cpp (Id)
D3.11 = 12. $CCCpqpp$ (Peirce)
D3.12 = 13. $CCpCpqCpq$ (Hilbert)
D10.2 = 14. $CCpqCCqrCpr$ (Syll)
D10DD14.2.14 = 15. $CCCprsCCqrCCpqs$
D73 = 16. $CCpCqCprCqCpr$
DD15.16.9 = 17. $CCpqCCpCqrCpr$ (Comm-Frege)
D10.17 = 18. $CCpCqrCCpqCpr$ (Frege)
D14.1 = 19. $CCpqrCqr$ (Syll-Simp)
D14.19 = 20. $CCCqrsCCCpqrs$