

## THE LOGIC OF THE SYNTHETIC A PRIORI

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Modal logic, which recognizes two kinds of truth, the analytic and the contingent, and the corresponding two kinds of falsity, is well suited to the logical needs of those philosophies which recognize precisely those four modal values; for example, the conceptual pragmatism of C. I. Lewis, logical empiricism, and, among earlier philosophies, that of Hume, who distinguished 'relations of ideas' and 'matters of fact', and that of Leibniz, who contrasted truths based respectively on the law of contradiction and the principle of sufficient reason.

But there are philosophies which recognize also, and insist upon the importance of, the *synthetically* necessary and the corresponding kind of falsity; for example, various forms of realism, phenomenology, and neo-Kantianism, and of course the philosophy of Kant himself.

The purpose of the present paper\* is to propose a six-valued calculus of propositions suited to the logical needs of those latter philosophies. Our procedure will be to adopt a standard system of modal logic and to add to it appropriately. From among the several closely related systems of modal logic we choose C. I. Lewis' S2, which he, the modern founder of modal logic, regarded as *the* System of Strict Implication, and which is strong enough for our purposes. He set it forth in considerable detail in [1], Chapter VI and Appendixes II and III, a presentation which will frequently be referred to in what follows in this paper.

We must, however, change the *readings* which Lewis ordinarily gave to his principal modal symbols. He usually read ' $\sim\Diamond\sim p$ ' as ' $p$  is necessary'; but, since we recognize two kinds of necessity, let us read it rather as ' $p$  is analytically necessary'. He usually read ' $\sim\Diamond p$ ' as ' $p$  is impossible'; but since we recognize two kinds of impossibility, the one associated with analytic necessity, the other with synthetic necessity, let us read it as ' $p$  is strictly impossible'—extending the use of his word 'strict'. Similarly,

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