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THEORIES OF TYPES AND ORDERED PAIRS

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It is generally taken for granted that the Modern Theory of Types with its strictly linear type hierarchy is slightly weaker than the Simple Theory of Types with its complicated branching one. The reason for this supposition is the alleged banishment of relations between objects of unlike type from the Modern Theory, and with them such classical results as Cantor's Theorem. To be sure, such heterogeneous relations can be simulated (and the banished theorems along with them) but the simulation is not quite satisfying because it blurs fundamental distinctions of type.[1]

The fact is that Wiener's original representation of ordered pairs, although only given for a few pairs of relative types, did not present such difficulty.[2] It has been, perhaps, the nearly universal adoption of Kuratowski's simpler construction that has blinded us to this fact. In what follows I propose a trivial modification of the Kuratowski construction which restores severed connections by allowing the possibility of true heterogeneous ordered pairs and hence also heterogeneous relations and all the classical results requiring their existence for proof. The point, then, is that the Modern Theory of Types is fully equivalent to the Simple Theory of Types.

In what follows 'Λ^{n+1}' will denote the class of all objects of type n which

1. For a brief description and comparison of these two theories in which the allegation is made and the usual method of simulation presented, see W. V. Quine, *Set Theory and its Logic*, revised edition (Belnap Press, Cambridge, Mass., 1971), pp. 249-265. For a more complete description of the Simple Theory of Types, see Hilbert and Ackermann, *Principles of Mathematical Logic* (Chelsea, New York, 1950), pp. 152-161.

2. Norbert Wiener, "A simplification of the logic of relations," *Proceedings of the Cambridge Philosophical Society*, vol. 17 (1914), pp. 387-390, reprinted in Jean van Heijenoort, *From Frege to Gödel* (Harvard University Press, Cambridge, Mass., 1967), pp. 224-227.

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