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UNIVERSAL PAIRS OF REGRESSIVE ISOLS

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1 *Introduction* Universal isols were first introduced by E. Ellentuck in [4] to provide a uniform source of counter-examples for proposed arithmetic statements in Λ. Prof. Ellentuck was also the first to prove, in unpublished notes, the existence of regressive universal isols, which provide a source for counter-examples in Λ_R; his proof is essentially a category argument. The present paper generalizes this argument to prove the existence of universal pairs of regressive isols which can serve as a source of counter-examples for proposed properties of Λ_R^2.

For f a recursive combinatorial function, let C_f denote the canonical extension of f to the isols; if f is recursive, then D_f denotes the canonical extension. From [4] we have the following definition: An isol T is *universal* if for each pair of recursive, combinatorial functions f and g,

$$C_f(T) = C_g(T) \rightarrow \{x \mid f(x) \neq g(x)\} \text{ is finite}$$
or
$$\text{there exists a number } n \text{ such that } x \geqslant n \rightarrow f(x) = g(x).$$

We are interested here in pairs of regressive isols (S, T) that have the property that if $f(x, y)$ and $g(x, y)$ are any recursive, combinatorial functions of x and y, then the identity $C_f(S, T) = C_g(S, T)$ will imply certain non-trivial similarities between the two functions f and g.

One analogue of the above definition would require a universal pair (S, T) of regressive isols to have the property that for $f(x, y)$ and $g(x, y)$ any recursive, combinatorial functions,

$$C_f(S, T) = C_g(S, T) \rightarrow \{(x, y) \mid f(x, y) \neq g(x, y)\} \text{ is finite.}$$

However, it is not difficult to construct recursive combinatorial functions $\tilde{f}$ and $\tilde{g}$ having the property that for all infinite regressive isols S and T,

$$C_{\tilde{f}}(S, T) = C_{\tilde{g}}(S, T) \text{ and } \{(x, y) \mid \tilde{f}(x, y) \neq \tilde{g}(x, y)\} \text{ is infinite;}$$

even easier functions refute the implication if S or T is taken to be finite. Thus we see that this analogue of the one-dimensional definition is too stringent, and we are led to the following definition: A pair of regressive

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