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# NONSTANDARD PROBABILITY

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*Introduction*[1] If F is a finite set, there is a natural probability measure on F given by $\mu_F(A) = \|A\|/\|F\|$, where $A \subseteq F$ and $\|X\|$ denotes the number of elements in a finite set X. In their paper [1], A. R. Bernstein and F. Wattenberg have shown the existence of a *finite set F such that if A is a Lebesgue measurable subset of [0, 1], then $\mu_F(A) = \|{}^*A \cap F\|/\|F\| \simeq m(A)$, where m is Lebesgue measure. The measure $\mu_F$ is called a sample measure. Since Lebesgue measure on [0, 1] is the measure induced by the uniform distribution on [0, 1], it is natural to ask for what other probability distributions on the real numbers can a similar result be shown. If the notion of sample measure is generalized, then the result of [1] may be extended to arbitrary real measures induced by probability distributions. This generalization and extension form the main portion of this paper. As an application of this extended result two nonstandard theorems of the central limit type are stated.

*Preliminaries* Let *R be an enlargement of the real number system R. Each set or concept in R will receive the prefix '*' when denoting the corresponding set or concept in *R, e.g., a *finite subset of *R is a subset on which there is an internal bijection onto an initial segment of *N, the enlargement of the natural numbers. (See [3] or [4] for more details.) Let $\langle z_n \rangle_{n \epsilon N}$ be a sequence (necessarily external) in *R. Then $\lim_n z_n = z$ will mean $z \epsilon R$ and, for every standard $\epsilon > 0$, there exists an $m \epsilon N$ such that $n \geq m$ implies $|z_n - z| < \epsilon$. Equivalently, $\lim_n z_n = z$ means the $z_n$ are eventually near-standard and z is the limit of the standard parts of these $z_n$.

A double sequence $\{\{X_{nk}\}\}$ will be a collection of random variables with $n \epsilon N$, $1 \leq k \leq k_n$, and $k_n \to \infty$ as $n \to \infty$. A double sequence $\{\{X_{nk}\}\}$ is said to

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