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COMPUTABILITY ON FINITE LINEAR CONFIGURATIONS

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In [1] H. Friedman posed the following problem: "... to fill in the blank in: Turing's operations are to finite linear configurations as are ________ to arbitrary finite configurations." For these purposes we make the following assumptions:

(1) The arbitrary finite configurations of members of $\mathcal{S}$ are the elements of the closure $\mathcal{S}^\#$ of $\mathcal{S}$ under finite set formation, i.e., $\mathcal{S}^\# = \bigcap\{\mathcal{U}: \mathcal{S} \subset \mathcal{U}$ and $\{x_1, \ldots, x_n\} \in \mathcal{U}$ whenever $x_1, \ldots, x_n \in \mathcal{U}\}$.
(2) The finite linear configurations of members of $\mathcal{S}$ are the elements of the closure $\mathcal{S}^*$ of $\mathcal{S}$ under formation of ordered pairs, i.e., $\mathcal{S}^* = \bigcap\{\mathcal{U}: \mathcal{S} \subset \mathcal{U}$ and $\langle x, y\rangle \in \mathcal{U}$ whenever $x, y \in \mathcal{U}\}$.

Such mathematical renderings of abstract concepts, e.g., "configuration" and "computability," must remain theses until such time as one accepts as evident enough mathematical properties of the notion involved to prove some sort of a characterization theorem on the basis of those properties. However, experience at translating mathematics into set theory seems to indicate that whatever "arbitrary finite configurations" and "finite linear configurations" are, reasonable representations for them can be found in $\mathcal{S}^\#$ and $\mathcal{S}^*$ respectively.

In [2], on the basis of a number of evident mathematical properties of computability, the notion of computability on $\mathcal{S}^\#$ is characterized in terms of computability on the natural numbers. Since $\mathcal{S}^*$ is a computable subset of $\mathcal{S}^\#$, we automatically get a characterization of computability on finite linear configurations. It is the purpose of this note to show that computability on arbitrary finite configurations is a strict generalization of computability on finite linear configurations in the sense of the following theorem.

Henceforth, we assume that $\mathcal{S}$ is infinite and free in the sense that the members of $\mathcal{S}$ are $\in$-minimal in $\mathcal{S}^\#$ (i.e., $s \cap \mathcal{S}^\# = \emptyset$ for all $s \in \mathcal{S}$).

Theorem *There is no embedding θ of $\mathcal{S}^\#$ into $\mathcal{S}^*$ such that for every partial function F on the range of θ, F is computable on $\mathcal{S}^*$ iff $\theta^{-1} F \theta$ is computable on $\mathcal{S}^\#$.*

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