

SINGLE AXIOMS FOR ATOMISTIC AND ATOMLESS MEREOLGY

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It is part of the folk-lore of the subject, that Leśniewski's mereology is neutral with respect to the existence of atoms. It has long been known that one could have a system of atomless mereology by adding the following axiom or its equivalent to any mereological axiom system.

$$[A]: A \varepsilon A \supset (\exists B). B \varepsilon \text{pr}(A).$$

Similarly one could have a system of atomistic (completely atomic) mereology by adding

$$[A]: A \varepsilon A \supset (\exists B): B \varepsilon \text{el}(A) : [C]: C \varepsilon \text{el}(B) \supset C = B.$$

If we define the name "atm" by

$$[B]: B \varepsilon \text{atm} \equiv B \varepsilon B : [C]: C \varepsilon \text{el}(B) \supset C = B$$

this simplifies to

$$[A]: A \varepsilon A \supset (\exists B). B \varepsilon \text{el}(A) . B \varepsilon \text{atm}.$$

Rickey, *cf.* [4], p. 90, introduced the functor "at" defined by

$$[AB]: B \varepsilon \text{at}(A) \equiv B \varepsilon \text{el}(A) . B \varepsilon \text{atm}$$

which further reduces the characteristic axiom of atomistic mereology to

$$[A]: A \varepsilon A \supset (\exists B). B \varepsilon \text{at}(A).$$

Using Rickey's functor "at" Sobociński axiomatized atomistic mereology in [4]. Lejewski gave the first single axioms for atomistic and atomless mereology in [2]. In this paper we shall give shorter single axioms for both systems.

Lejewski's single axiom for atomistic mereology is

$$\begin{aligned} L1 \quad [AB] &:: A \varepsilon \text{at}(B) \equiv B \varepsilon B :: [CDa]:: [E]: E \varepsilon C \equiv [F]: F \varepsilon \text{at}(E) . \\ &\equiv [\exists G]. F \varepsilon \text{at}(G) . G \varepsilon a :: D \varepsilon \text{at}(B) . B \varepsilon a \supset \text{at}(A) \varepsilon A . A \varepsilon \text{at}(C) . \end{aligned}$$

This contains ten occurrences of ε . We shall show that the shorter (nine occurrences of ε) proposition