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DECOMPOSABLE ORTHOLOGICS

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1 *Introduction* The inability of Boolean algebras to represent the logic of quantum mechanics, as observed by Birkhoff and von Neuman [1], Mackey [11], and others [6], [12], [13], has created interest in logics which represent the propositions of empirical science affiliated with more than one physical operation.

In [4] and [5] Foulis and Randall introduced the concept of a *manual* of physical operations whose elements are identified with the sets of outcomes for physical experiments. The logic affiliated with such a manual is called an *orthologic*. Most, if not all, of the logics which have been proposed for quantum mechanics by the above authors have been orthologics.

In [2], Dacey introduced the *Dacey sum* of a collection of manuals, which is a manual whose elements are identified with the outcome sets of two stage physical experiments. A manual is said to be *indecomposable* if it cannot be written in a non-trivial fashion as a Dacey sum. In [8] the author defines the *composite product* of a family of orthologics, which corresponds to the logic of the Dacey sum of manuals whence the orthologics came. Here we define an orthologic to be *indecomposable* if it cannot be written in a non-trivial fashion as a composite product of orthologics. Our main theorem states that an orthologic is indecomposable if and only if every member of a wide class of manuals whence it came is indecomposable. Our definition of decomposability of an orthologic appears to be the natural generalization of the concept of irreducibility in lattice theory. In particular, we also show that every indecomposable orthologic is irreducible.

The majority of the results appearing here may be found in the author's dissertation, submitted to the graduate school of the University of Massachusetts in 1972, and written under the direction of Professor D. J. Foulis. The author would like to thank Professor Foulis for his assistance in this work.

2 *Definitions and Motivation* Let $\mathfrak{A}$ denote a non-empty set of non-empty sets. Write $A = \bigcup \mathfrak{A}$. For $x, y \in A$, write $x \perp y$ to mean that $x \neq y$ and there

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