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IMPLICATION CONNECTIVES IN ORTHOMODULAR LATTICES

L. HERMAN, E. L. MARSDEN, and R. PIZIAK

1 *Introduction* It was pointed out as long ago as 1936 by Birkhoff and von Neumann [2], that the logic of empirically verifiable propositions about a quantum mechanical system is not classical. That is, quantal propositions do not tend to band together to form a Boolean algebra. The distinguishing feature of quantum mechanics, namely the existence of quantities which are not simultaneously measurable, led to an attack on the *distributive law* as the law of logic which is least tenable in quantum logic. Though Birkhoff and von Neumann argued in favor of the weaker *modular law*, subsequent researchers have rejected modularity in favor of the yet more general *orthomodular law*. Thus, the basic assumption of the quantum logic approach to quantum mechanics today is that the empirically verifiable propositions relevent to a given physical system form an orthomodular lattice (see Jauch [17], Varadarajan [36]). Some have objected that to demand a lattice is already too much. The appropriate structure for quantum logic then becomes an orthomodular partially ordered set (see Mackey [26], Pool [32]). However, we shall ignore this controversy here and maintain a lattice structure throughout.

The assumption of conventional quantum mechanics is that the logic of a physical system is modeled by the orthomodular lattice of all closed subspaces of a complex separable infinite dimensional Hilbert space, or, equivalently, by the lattice of all projection operators on the Hilbert space (see von Neumann [37], n.b. Projections as Propositions, p. 247 ff.). One of the purposes of the quantum logic approach to quantum mechanics has been to elucidate this somewhat ad hoc assumption. More recently, Foulis and Randall [9, 10, 33] have shown that orthomodular lattices lie at the heart of their "operational statistics". This should help to clarify the logical foundations of any empirical science.

Our paper begins with a brief description of quantum logic and the problem of introducing an implication in such a logic. Next, several candidates for such an implication are studied. Our point of view then becomes axiomatic as we abstract what we consider to be the essential features of an internal implication connective. With the help of a given

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