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SOME EXTENSIONS OF S3

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Sobociński [4] obtained modal systems S3.02, S3.03, S3.04, by adding to S3 the respective axioms

Ł1 $((p \prec \mathsf{L}p) \prec p) \prec (\mathsf{LML}p \supset p)$
Ł2 $((p \prec \mathsf{L}p) \prec p) \prec (\mathsf{LML}p \prec p)$
L1 $\mathsf{LML}p \prec (p \supset \mathsf{L}p)$.

This note (in the notation of [2]) clarifies the relationships of these systems to one another, and to other extensions of S3.

It is easy to test ([2], p. 279f.) that

(1) $\vdash_{S3} (\mathsf{L}p \prec (\mathsf{L}q \supset r)) \prec (\mathsf{L}p \prec (\mathsf{L}q \prec r))$.

A substitution instance of this is **Ł1** $\prec$ **Ł2**, so S3.02 = S3.03. It is also easy to test ([2], p. 284f.) that $\vdash_{S3.5}$**Ł1** and $\vdash_{S3.5}$**L1**. Hence both S3.02 and S3.04 are contained in S3.5. Moreover, both S3.02 and S3.04 contain the system 16s of [3]. For:

S3: (2) $((\mathsf{L}p \supset q) \prec \mathsf{L}r) \prec (\mathsf{L}s \supset \mathsf{LL}s)$

Ł2$[p/(\mathsf{L}p \supset \mathsf{LL}p)]$, (2) $[q/\mathsf{LL}p, r/(\mathsf{L}p \supset \mathsf{LL}p), s/p]$:

(3) $\mathsf{LML}(\mathsf{L}p \supset \mathsf{LL}p) \prec (\mathsf{L}p \supset \mathsf{LL}p)$

(3), (1): (4) $\mathsf{LML}(\mathsf{L}p \supset \mathsf{LL}p) \prec (\mathsf{L}p \prec \mathsf{LL}p)$

S3: (5) $\mathsf{LML}(\mathsf{LML}p \supset \mathsf{L}p)$

(4)$[p/\mathsf{MLLM}p]$, (5) $[p/\mathsf{LM}p]$, S3: (6) $\mathsf{LMLLM}p \prec \mathsf{LLM}p$

Hence ([3], p. 275) S3.02 contains 16s. Also:

S3: (7) $\mathsf{LMLLM}p \prec \mathsf{LM}p$

L1 $p/\mathsf{LM}p$, (7), S2: (8) $\mathsf{LMLLM}p \prec \mathsf{LLM}p$

Hence S3.04 contains 16s.

Another system between S3.5 and 16s is 14r ([3], p. 273). But Table 2.2 ([3], p. 274) (i.e., Lewis Group II) readily shows that neither S3.02 nor S3.04 contains 14r, so these systems have 16s modalities. Sobociński has pointed out ([4], p. 417) that S3.02 does not contain S3.04. Also, by Table 3.2

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