Notre Dame Journal of Formal Logic
Volume XVI, Number 2, April 1975
NDJFAM

CONCRETE COMPUTABILITY

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On the basis of several intuitively obvious properties of computability we show, among other things, that F is a computable partial function on the closure $\mathcal{S}^\#$ of an infinite set $\mathcal{S}$ under finite set formation iff there exists a countably infinite subset $\mathcal{U}$ of $\mathcal{S}$ and a finite subset $\mathcal{T}$ of $\mathcal{U}$ such that

(1) if g is any permutation on $\mathcal{S}$ leaving the members of $\mathcal{T}$ fixed, then $Fg^\# = g^\# F$ where $g^\#$ denotes the canonical extension of g to the members of $\mathcal{S}^\#$,
(2) if θ is any bijection from $\mathcal{U}$ onto the even numbers then $\overline{\theta} F \overline{\theta}^{-1}$ is computable on $\mathbf{N}$ where $\overline{\theta}$ is the unique extension of θ such that if $y_1, \ldots, y_n \in \mathcal{U}^\#$,

$$\theta(\{y_1, \ldots, y_n\}) = 2(2^{y_1} + \ldots + 2^{y_n}) + 1.$$

1 *Introduction* The *classical* theory of computability is concerned with *finitely* long processes on certain *finitary* or concrete combinations of objects from a *finite* generating set using a *finite* amount of *a priori* information. Church has suggested a certain mathematical (i.e., set theoretic) definition for this abstract concept. This suggestion is called *Church's Thesis*. Various others, e.g., Turing, Post, Markov, have made similar suggestions which have been shown equivalent to that of Church. Interesting generalizations have been obtained by extending any or several of these finitary aspects of computability to the infinite case. The theory of *concrete computability* is that generalization obtained by allowing the generating set to have arbitrary cardinality. This study can be motivated by considering such questions as "In what sense are the rules for first-order logic on uncountably many relation symbols effective?" Many authors have suggested definitions of computability that apply in such cases, e.g., Montague [2] and Moschovakis [3]. In this paper we characterize concrete computability in terms of classical computability, on the basis of several intuitively obvious properties of computability. These results together with Church's Thesis give an absolute characterization of concrete computability.

First version received November 9, 1972.
Final version received May 2, 1974.