

AXIOMATIZATION OF FRAGMENTS OF S5

THOMAS W. SCHARLE

The principal result of this paper* is that the well-formed formulas of the system S5 of modal logic can be expressed in a kind of normal form which is restricted in a certain way. That normal forms exist for S5 is well known, but the results given here indicate that the normal forms can be expressed largely—in a sense to be made clear later—using only implication and necessity. From this result, it then follows that there is a uniform pattern for axiomatizing those functionally incomplete parts of the system S5 which contain at least implication and necessity. These functionally incomplete parts are the 'fragments' of the title.

The principal result, in turn, follows from the discovery of a sequence of expressions in S5, containing only implication and necessity, having certain welcome properties. That such functors might exist was suggested by the results of Canty and Scharle, and Massey, mentioned below. In order to find the expressions, the next step was to use a digital computer to search, in effect, all possibilities in hope of finding, in a reasonable amount of time, a successful match. Fortunately, many matches were found. This is one of the few published cases in which a digital computer has been used to find a solution to a non-trivial problem in logic.

Having found a solution for the two-variable case this way (the case for one variable is trivial), one sees that the next step is either to use the computer again, or to proceed by more conventional methods. To use the computer again, it turns out, would involve extremely long computation times, so this was not attempted. But, as will be shown in this paper, an analytical approach shows at least the existence of solutions in general.

*This paper is adapted from a thesis written under direction of Professor Bolesław Sobociński and submitted to the Graduate School of University of Notre Dame, in partial fulfillment of the requirements for the degree of Doctor of Philosophy with Philosophy as the major subject in August, 1973. I wish to thank Professor Bolesław Sobociński for his help and encouragement in the writing of this paper, the Computing Center of West Virginia University for use of the computing facilities, and Mr. Addison Fischer for programming assistance.