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## LOGIC OF ANTINOMIES

## F. G. ASENJO and J. TAMBURINO

Purpose There are essentially three ways of looking at antinomies. The first is to consider them as undesirable anomalies. This is the predominant view, and was Russell's when he discovered the famous antinomy that bears his name. Because of the devastating consequences that result from the presence of a single antinomy in any system based on classical logic, this view has been understandably strong. It is well known, for example, that Frege reacted with extreme and lasting consternation to Russell's discovery. The second view considers antinomies less dramatically, taking them as merely harmless abnormalities. Remarkably enough, this was Cantor's position ([4], pp. 384-5), as well as Wittgenstein's: "If a contradiction were now actually found in arithmetic-that would only prove that an arithmetic with such a contradiction in it could render a very good service" ([5], p. 181e). From this second point of view the problem with antinomies is how to confine them, how to prevent their turning every well-formed formula into a theorem without eradicating them and without abandoning or radically altering the system in question. The third and last view is to see antinomies as useful logical entities.<sup>1</sup> According to this position antinomies must be integrated into logical systems starting with the propositional calculus, bearing in mind that while some sentences have only one truth value, others have two. To use an example from ordinary language, a sentence such as "It is raining here now" can only be true or false, but not both. In contrast, "Peter is a good man" is not a singlevalued sentence and no attempt should be made to make it one. There are a number of more or less artificial interpretations of propositional semantics whose chief objective is to suppress the antinomies' first obvious meaning, be the antinomies taken from ordinary language or from mathematics. For example, it is against Cantor's original conception of

<sup>1.</sup> There are several philosophic precedents to this position. Kant, for example, attached to antinomies the positive function of preventing reason from slumbering —apart from their playing a very important demonstrative role in his own philosophy.