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ORDERED PAIRS AND CARDINALITY IN NEW FOUNDATIONS

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In any set theory, two sets are said to have the same cardinality if there is a bijection between them. Thus the notion of having the same cardinality (which we shall call 'being equipollent') is dependent on that of function and hence on that of ordered pair. We shall show that in Quine's set theory **NF** (as formulated in [3], for instance) the definition of ordered pair which is used affects whether, or not, two sets are equipollent, and we make some further considerations based on this fact.* The following definitions are made to aid our discussion, and we hope that it is obvious how they could be made precise.

Definition 1 A formula $\psi(x, y, z)$ with exactly three free variables is said to represent an ordered pair relation in a set theory T if

(i) $T \vdash \forall x, y \exists ! z \psi(x, y, z)$,

and

(ii) $T \vdash \forall x, x', y, y', z[(\psi(x, y, z) \wedge \psi(x', y', z)) \rightarrow (x = x' \wedge y = y')]$.

Definition 2 If ψ represents an ordered pair in a set theory T, then $x \approx_\psi y$ is a formula which, in a natural way, says that there is a function, represented as a set of ordered pairs which are defined using ψ, which is a bijection from x to y.

We will always assume that $z = \langle x, y\rangle$ is a formula which says that z is the Kuratowski ordered pair (i.e., $\{\{x\}, \{x, y\}\}$) and then in both **ZF** and **NF** this represents an ordered pair relation. Also, $x \approx y$ will always be $x \approx_\phi y$, where ϕ is $z = \langle x, y\rangle$.

The next theorem shows that, in a certain sense, the notion of being equipollent is independent of the representation of ordered pairs in **ZF** set theory.

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