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TRANSLATION OF THE SIMPLE THEORY OF TYPES INTO A FIRST ORDER LANGUAGE

H. JULIAN WADLEIGH

1 *Introduction* In this paper we formulate (1) a simple theory of types, $\mathfrak{F}\omega$, (2) a first order language with postulates, $\mathfrak{F}(t)$, and (3) a set of rules for translating $\mathfrak{F}\omega$ into $\mathfrak{F}(t)$. We prove that a wff of $\mathfrak{F}\omega$ is a theorem of $\mathfrak{F}\omega$ if and only if its translation is a theorem of $\mathfrak{F}(t)$.

The set of postulates of $\mathfrak{F}(t)$ is a modification of the set of formulas D_{11}-D_9 in Hintikka's [4],[1] the main difference being that Hintikka's D_8, which contains a quantified predicate, is replaced by a series of postulates without quantified predicates. Thus we reduce type theory to a first order language where Hintikka's reduction was to a second order language. There is also a difference in approach, this paper being concerned exclusively with syntax, while Hintikka gives considerable attention to model theory.

$\mathfrak{F}(t)$ can be used to talk about the individuals and predicates of type theory in much the same way as we talk about sets in axiomatic set theory. The postulates of $\mathfrak{F}(t)$, which in their intended interpretation assert the existence of the individuals and predicates of $\mathfrak{F}\omega$ and describe their relations to each other, are roughly analogous to the axioms of set theory. We do not suggest that $\mathfrak{F}(t)$ be used to prove results that can be proved in type theory. In doing so one would lose the simplicity and directness of type theory and its capacity to reproduce the structure of intuitive mathematical thinking—a virtue not possessed by any of the popular brands of axiomatic set theory. More promising is the use of $\mathfrak{F}(t)$ to talk about the symbols and syntax of $\mathfrak{F}\omega$. For this purpose one could extend $\mathfrak{F}(t)$ by introducing predicate variables (including those of higher types) and applying quantifiers to such variables. Such an extension of $\mathfrak{F}(t)$ would be a formalized metalanguage of type theory. But this is beyond the scope of the present paper, which is to exhibit a reduction of type theory to a first order language.

1. [4], p. 84.

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