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LOGIC WITHOUT TAUTOLOGIES

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In the first edition of *Introduction to Logic* (p. 259), Copi gave a system of natural deduction for sentential calculus, and he included in the second edition of *Symbolic Logic* (pp. 53 ff.) my proof that the system is incomplete. In this paper, I want to show, first, that the matrix used to prove the incompleteness of the system in fact furnishes a decision procedure for it; second, that any "formal" extension of the system is complete; third, that the system contains no "tautologies" or "theorems", though it contains "contradictions"; fourth, that though the system does not permit the deduction of all conclusions from premisses which tautologically imply them, still it does permit the deduction of some tautological equivalent of any non-tautological conclusion tautologically implied by the premisses. Another result may also be of interest. The usual replacement rule does not hold for the system, although a certain "weak" replacement rule does hold. Finally, the system actually worked with, proved equivalent to Copi's, is perhaps interesting in its own right.

1 *The equivalence of Copi's system and* **C**. The rules of Copi's system are here transcribed in the metalinguistic notation that will be used throughout this paper. Thus, instead of "p" and "q" and the like, Roman capitals, with or without subscripts and other affixes, are used as metalinguistic variables. (In one later context, however, "A" and "B" are used as proper names of atomic sentences.) In the presentation of Copi's system, "$\vdash_C$" will mean "yield(s) by Copi's rules" and "$\underset{C}{\Leftrightarrow}$" will mean "may, according to Copi's rules, replace or be replaced by".

The first nine of Copi's rules are:

1. Modus Ponens: $S_1 \supset S_2, S_1 \vdash_C S_2$
2. Modus Tollens: $S_1 \supset S_2, \sim S_2 \vdash_C \sim S_1$
3. Hypothetical Syllogism: $S_1 \supset S_2, S_2 \supset S_3 \vdash_C S_1 \supset S_3$
4. Disjunctive Syllogism: $S_1 \vee S_2, \sim S_1 \vdash_C S_2$
5. Constructive Dilemma: $(S_1 \supset S_2) \cdot (S_3 \supset S_4), S_1 \vee S_3 \vdash_C S_2 \vee S_4$
6. Destructive Dilemma: $(S_1 \supset S_2) \cdot (S_3 \supset S_4), \sim S_2 \vee \sim S_4 \vdash_C \sim S_1 \vee \sim S_3$
7. Simplification: $S_1 \cdot S_2 \vdash_C S_1$

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