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NORMAL AND SKEW SYSTEMS

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Preface Suppose the formal system T has a binary connective $\rightarrow$. If T has:

$$\mathbf{MP}: \frac{A, A \rightarrow B}{B}$$

among its rules of inference, it is natural to ask: "Which theorems of T are still provable (without repetitions) if one refuses to use **MP** except when A precedes $A \rightarrow B$?" Roughly speaking, we will say that T is normal if the answer to this question is "All of them."

In forming a precise definition a certain difficulty becomes apparent. One might be able to augment the sequence $\ldots, A \rightarrow B, \ldots, A, \ldots, B$ by new formulas to obtain $\ldots, A \rightarrow B, \ldots, A, \ldots, C, \ldots, C \rightarrow B, \ldots, B$, and thus avoid the issue. For example, if T had the axiom schemes:

A1) $A \rightarrow (B \rightarrow A)$
A2) $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$,

then $\ldots, A \rightarrow B, \ldots, A, \ldots, B$ could be replaced by:

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$A \rightarrow B$
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A
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.
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F
$A \rightarrow (F \rightarrow A)$ A1
$F \rightarrow A$ **MP**
$(A \rightarrow B) \rightarrow (F \rightarrow (A \rightarrow B))$ A1
$F \rightarrow (A \rightarrow B)$ **MP**
$(F \rightarrow (A \rightarrow B)) \rightarrow ((F \rightarrow A) \rightarrow (F \rightarrow B))$ A2
$(F \rightarrow A) \rightarrow (F \rightarrow B)$ **MP**
$F \rightarrow B$ **MP**

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