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VARIOUS SYSTEMS OF SET THEORY BASED ON COMBINATORY LOGIC

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1. *Introduction* We consider in this paper various systems of set theory which, in their usual formulation, are based on predicate calculus. Here the predicate calculus is an applied predicate calculus, in that alongside the various categories of variables we have constants of various categories; concerning these constants we assume certain additional axioms.

In order to translate this situation into combinatory logic the axioms we assume must be such as to make each of these new constants satisfy the proper grammatical condition (see [2]). If that is the case, such constants can be substituted for the variables with the same grammatical conditions, and these grammatical conditions can be detached from the set of premises of such formulas. Sometimes, these constants can be defined in such a way that the only essentially new axiom is that giving the grammatical condition.

One of these constants is the ob $\mathbf{A}$, this is the range of obs over which an ob must be a propositional function before we can apply the deduction theorem (see [3]); we shall also take it as our range of quantification. Thus in this chapter we apply the theorems of [2] with $\mathbf{A}$ for a. The condition $\mathbf{L}a$ can be dropped in these theorems as we have $\vdash \mathbf{LA}$ by Axiom 8 of [2].

Next we consider the relation ε which is usually taken as primitive in set theory. In our system ε can be defined by $\mathbf{CI}$ so that $x \varepsilon y$ is translated as yx. The ob $\mathbf{CI}$ can be proved to be a predicate as we have in each system an axiom which gives us

$$\mathbf{A}u,\ \mathbf{A}v \vdash \mathbf{H}(uv).$$

This rule, as was shown in [4], is inconsistent with $\mathbf{E}$ for $\mathbf{A}$, but with a suitably restricted $\mathbf{A}$ there should be no problem. The rule gives us $\mathbf{A}u$, $\mathbf{A}v \vdash \mathbf{H}(\mathbf{CI}vu)$ which by the deduction theorem gives us,

$$\vdash \mathbf{F}_2\mathbf{AAH}(\mathbf{CI}).$$

Other constants that we shall be using are the first order predicates $\mathbf{M}$ and $\mathbf{M}_1$, where $\mathbf{M}$ is a new primitive for the category of all sets, and $\mathbf{M}_1$ is

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