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CONCERNING THE PROPER AXIOMS OF S4.02

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In [4] it has been established that the addition of the following formula

Ł1 $\quad$ *ℭℭℭpLppCLMLpp*

as a new axiom, to S4 generates a system, called S4.02, which is a proper extension of S4. And obviously, *cf.* [6], in the field of S4, **Ł1** is inferentially equivalent to

Ł2 $\quad$ *ℭℭℭpLppLCLMLpp*

In this note it will be shown that in the field of S4 each of the following two formulas

Ł3 $\quad$ *ℭℭℭpLpLpCLMLpLp*

and

Ł4 $\quad$ *ℭℭℭpLpLpCLMLpp*

is inferentially equivalent to **Ł1**.

Proof:

1 Assume S4 and **Ł3**. Then, obviously, we have **Ł4**. Now, S4 yields the following formulas:

Z1 $\quad$ *ℭLpLLp*
Z2 $\quad$ *ℭℭpqℭLpLq*

Whence,

Z3 $\quad$ *ℭℭLℭpLpLpCLMLpp* $\quad$ [**Ł3**; *Z1*]
Ł1 $\quad$ *ℭℭℭpLppCLMLpp* $\quad$ [*Z2, p/ℭpLp, q/p*; *Z3*; S1°]

Thus, in the field of S4: {**Ł3**} → {**Ł4**} → {**Ł1**}.

2 Now, let us assume S4 and **Ł1**. Then:

Z1 $\quad$ *ℭℭvℭqrℭℭℭprsℭvℭℭpqs* $\quad$ [S4]
Z2 $\quad$ *ℭℭpqℭℭvℭℭprsℭvℭℭpqs* $\quad$ [S4]

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