

## GENERALISED LOGIC

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1. The logically unsophisticated will often protest when those who are more sophisticated draw out the consequences of their statements. If the discussion is pursued, it may be found that they are resistant to accepting the excluded middle law. Moreover, they may go further, by refusing to concede that a certain statement is or is not true and refusing to concede that it is or is not false, though admitting that it cannot be both true and false. Those who are more sophisticated have a powerful armoury for avoiding such pitfalls. This includes the adjectives of degree and emphasis and such devices as "partly the one thing and partly the other," "true or false but we don't know which," "true in one sense and false in another," "classes versus criteria," "extension versus intension" and in the last resort "neither true nor false" and "too vague to mean anything."

The one thing that is not attempted is to take the unsophisticated seriously, that is to say, to attempt to construct a logic in which there is a middle term that is not necessarily incompatible with truth or with falsity, though these remain incompatible with one another. In the present paper it will be shown that, contrary to expectation, such a logic can be constructed and that it is an interesting and very radical generalisation of elementary logic.

Take the matter from another point of view, by considering existing systems that modify or abrogate the excluded middle law. On the one hand, there are systems such as those of Heyting [1] and of Fitch [2] in which  $A \vee Np$  is not a theorem, but in which there is no third term " $?p$ ". On the other hand, there are such systems as those discussed by Rosser and Turquette [3], in which there are three or more terms, but in which the terms are incompatible with one another, in the sense that any proposition takes one and only one value. Comparing these two groups of systems, it may be asked whether a system could be constructed that, unlike the systems of the first group, includes a third or middle term, but unlike the systems of the second group is such that the middle term is not incompatible with  $p$  or with  $Np$ . The system of the present paper is such a system.