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THE GEOMETRY OF SOLIDS IN HILBERT SPACES

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In [6] A. Tarski presents a definition of concentric solids in Leśniewski's deductive system of Mereology [1, 2, 5]. If solid is interpreted as sphere in Euclidean geometry, Tarski showed that the set of equivalence classes of spheres determined by the definition of concentric corresponds to the set of points of the Euclidean space. Thus Mereology could be used as a foundation for Euclidean geometry. In this paper* we shall show that the above correspondence still holds in the case where the space is taken to be a real separable Hilbert space of dimension ≥ 2.

The definition of concentric presented below is based on the primitive relation of Mereology, "A is part of B," where A and B are restricted to be instances of the name solid. In a Hilbert space $\mathfrak{H}$ this relation will be interpreted as the sphere A is a subset of the sphere B where a sphere is a set of the form $\{\mathbf{x}/\|\mathbf{x} - \mathbf{a}\| < r\}$ with $\mathbf{a} \in \mathfrak{H}$, $r > 0$, and $\| \ \|$ denoting the norm of $\mathfrak{H}$.

We begin with the development of Tarski's definition of concentric. We note that the definitions below will be formulated using only the primitive relation of Mereology and definitions already given.

Definition 1 A *is disjoint from* B iff A and B are solids and whenever X is part of A then X is not part of B. (We note that for open balls in a Banach space this definition is equivalent to the statement that A and B are disjoint sets.)

Definition 2 A *is externally tangent to* B iff (i) A is disjoint from B and (ii) if A is part of X and is part of Y with B disjoint from X and disjoint from Y then X is part of Y or Y is part of X.

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